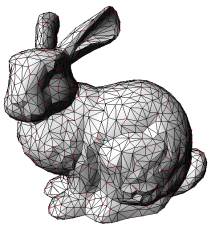
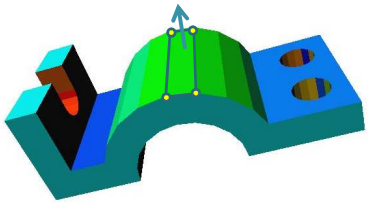




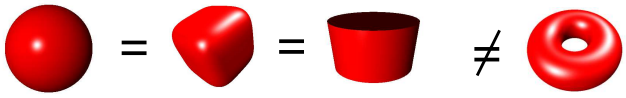
3.5 Représentation frontière

Boundary representation (Brep)

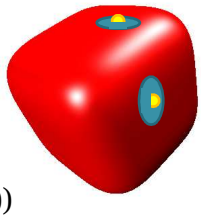
- Topologie (polyèdres faces planes)
  - Géométrie (des faces)

Frontière – aspect topologiques

- Arête, sommet = partition d'une frontière,
- Homéomorphisme … Cube ⇔ Sphère



- Frontière de S :



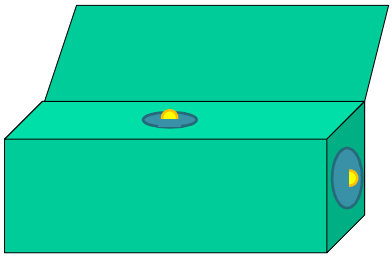
Fr(S) = variété de dimension 2

$$Fr(S) = Adh(S) - Int(S)$$

$$Fr(S) = Adh(S) \cap Adh(Comp(S))$$

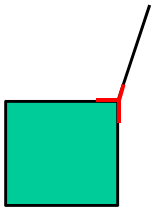
$$\forall P \in Fr(S), \forall Vois(P), Vois(P) \cap S \neq 0 et Vois(P) \cap Comp(S) \neq 0$$

Homogénéïté, régularité, ***frontière manifold***



Objet non régulier en 3D

Frontière non 2-manifold



Objet non régulier en 2D

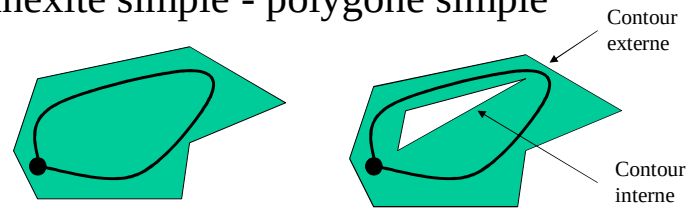
Frontière non 1-manifold

Exemples : MAYA, MeshLab

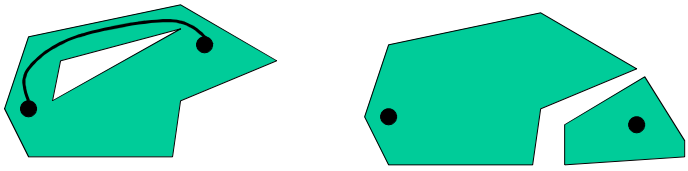
Connexité (Aspects topologiques)

lacets non-contractiles

- Connexité simple - polygone simple

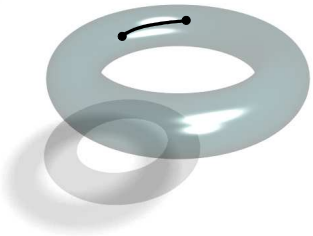


- Connexité par arc - composante connexe

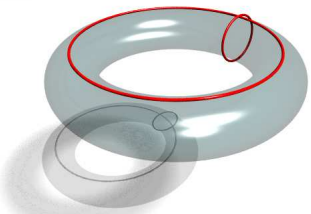


Connexité et trous

La frontière du tore est t-elle connexe ?



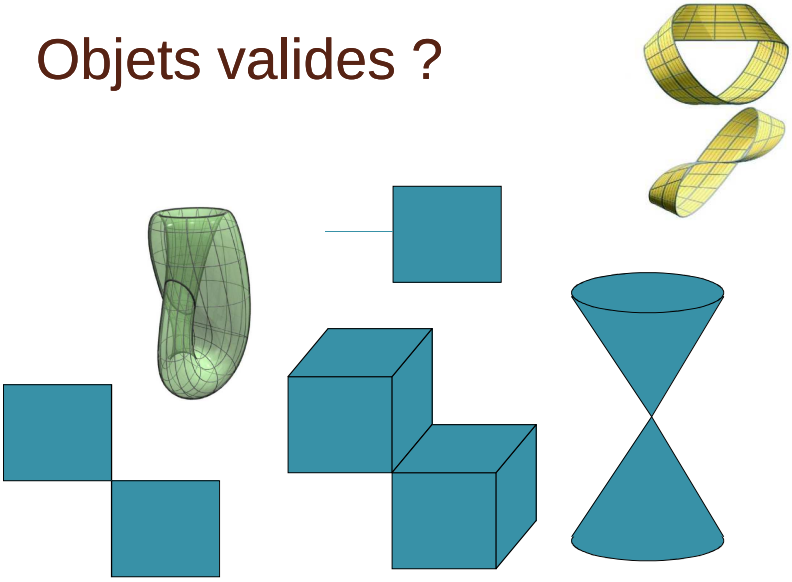
Connexe par arc



Non simplement connexe

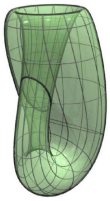
Existence de 2 familles de cycles non réductibles et indépendants

Objets valides ?

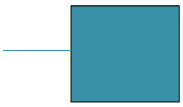


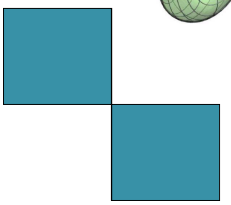
Objets valides ?

non orientable
non 2-manifold

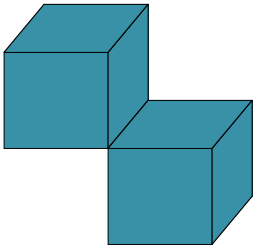


Non régulier

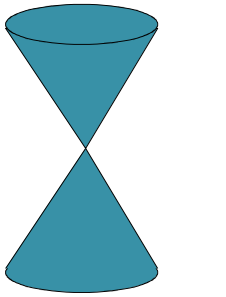




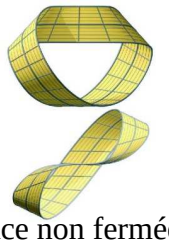
non 1-manifold



non 2-manifold



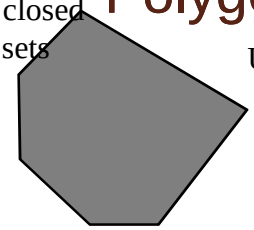
non 2-manifold



Surface non fermée

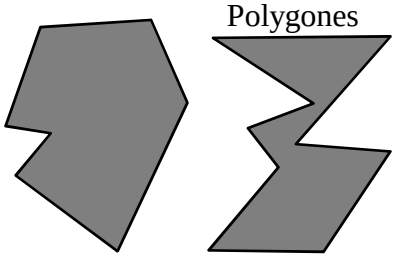
Polygones (Polyèdres)

compact closed
manifold sets



U : union

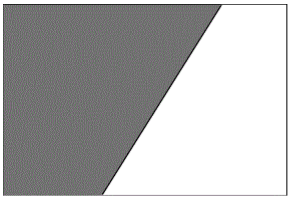
U



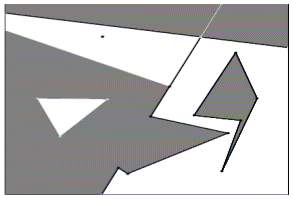
Polygones

Convexes polytopes

I



I, C

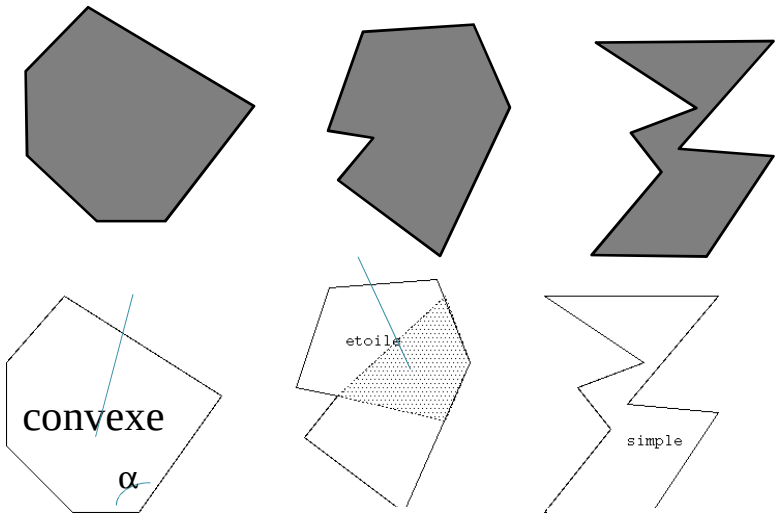


½ espaces ouverts

Nef-Polygones

I,C : Intersection et Complément de ½ espaces ouverts

Convexité



α <= 180°

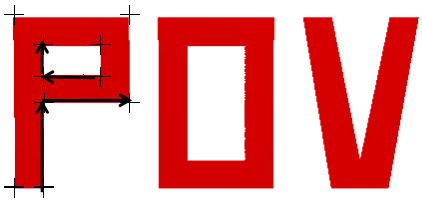
étoilé

simple

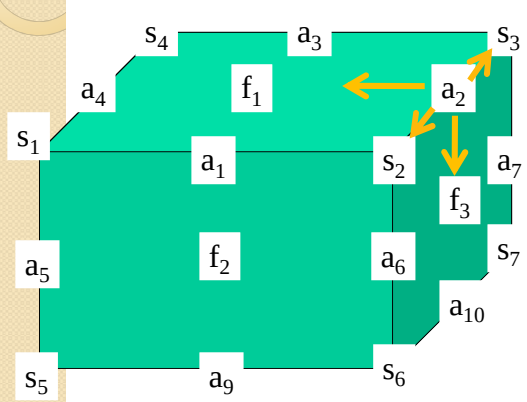
PovRay : polygones

PovRay

Polygon { nb_points
< x_{11},y_{11} >, … , <, x_{1i},y_{1i} > … < x_{11},y_{11}>,
< x_{21},y_{21} >, … , <, x_{2i},y_{2i} > … < x_{21},y_{21}>,
< x_{j1},y_{j1} >, … , <, x_{ji},y_{ji} > … < x_{j1},y_{j1}>
}



Polyèdres



Card(F) – Card(A)+Card(S)=2

Opérateur “bord” :

b(v) = {f_1, f_2…f_6}
b(f_3) = {a_2, a_6, a_{10}, a_7}
b(a_6) = {s_6,s_2}

Mais aussi incidence :
i(a_2) = {f_1, f_2}
…
Sommets d’un graphe :
F = { f_1,f_2…f_6}
A = { a_1, a_2…a_{12}}
S = { s_1,s_2…s_8 }

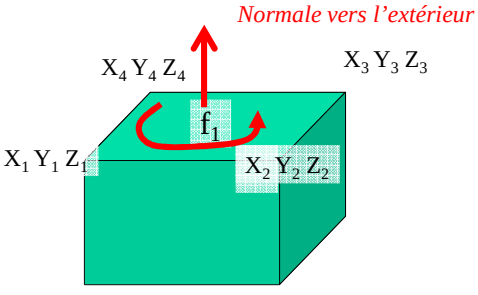
Description des polyèdres

Description suffisante (utilisateurs: WRL, X3D, Inventor)…

```
#VRML V2.0 utf8
…
Geometry  IndexedFaceSet {
ccw TRUE
solid TRUE
convex TRUE
coord Coordinate { point [
# Coordonnees des points
X1 Y1 Z1
X2 Y2 Z2
…
]}
coordIndex [
# Les faces
1 2 3 4 -1
…
]
```

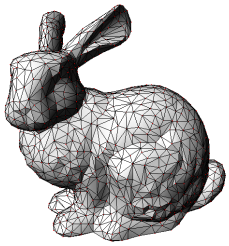
n’affiche qu’un coté des faces

les faces !



Face-based data structure
(~~edge-based data structure~~)

Formats …



STL

```
solid name
…
facet normal ni nj nk
outer loop
vertex v1x v1y v1z
vertex v2x v2y v2z
vertex v3x v3y v3z
endloop
endfacet
…
endsolid name
```

OFF

```
OFF
# cube.off
# A cube
8 6 12
1.0 0.0 1.0
0.0 1.0 1.0
-1.0 0.0 1.0
0.0 -1.0 1.0
1.0 0.0 -1.0
0.0 1.0 -1.0
-1.0 0.0 -1.0
0.0 -1.0 -1.0
4 0 1 2 3
4 7 4 0 3
4 4 5 1 0
4 5 6 2 1
4 3 2 6 7
4 6 5 4 7
```

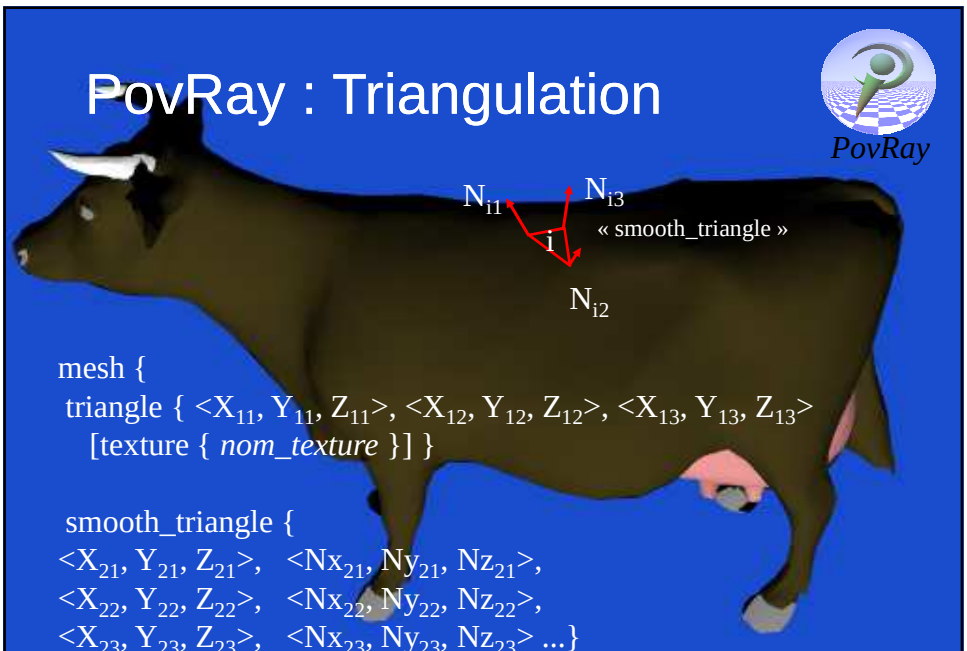
OBJ

```
# cube.obj
# list of vertices
v  1.0 0.0 1.0
v  0.0 1.0 1.0
v  -1.0 0.0 1.0
v  0.0 -1.0 1.0
v  1.0 0.0 -1.0
v  0.0 1.0 -1.0
v  -1.0 0.0 -1.0
v  0.0 -1.0 -1.0
…
# list of faces
f  1 2 3  4
f  8 5 1  4
…
# mais aussi :
# f  node/norm/texture
f 6/4/1 2/5/3 7/6/8 6/4/5
```

WRL

```
#VRML V2.0 utf8
…
Geometry  IndexedFaceSet {
# optionnel mais utile :
ccw TRUE
solid TRUE
convex TRUE
coord  Coordinate {
point [
# Coordonnees des points
X1 Y1 Z1
X2 Y2 Z2
…
]}
coordIndex [
# Les faces
1 2 3 4 -1
5 6 2 1 -1
…
]
}
```


PovRay : Triangulation



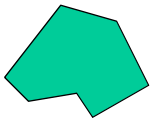
mesh {
triangle { <X_{11}, Y_{11}, Z_{11}>, <X_{12}, Y_{12}, Z_{12}>, <X_{13}, Y_{13}, Z_{13}>
[texture { *nom_texture* }] }

smooth_triangle {
<X_{21}, Y_{21}, Z_{21}>, <Nx_{21}, Ny_{21}, Nz_{21}>,
<X_{22}, Y_{22}, Z_{22}>, <Nx_{22}, Ny_{22}, Nz_{22}>,
<X_{23}, Y_{23}, Z_{23}>, <Nx_{23}, Ny_{23}, Nz_{23}> ...}
…}

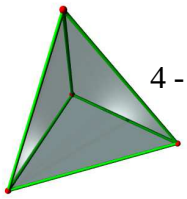
Fichier : cow2.pov

La relation d ’Euler-Poincaré

- dD : $\sum_{k=0}^{d-1}((-1)^k n_k(P)) = 1 - (-1)^d$ polytope
- 2D : -a + s = 0
- 3D : f - a + s = 2

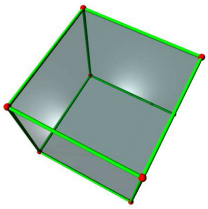


7 - 7 = 0



4 - 6 + 4 = 2

simplexe



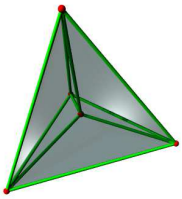
6 - 12 + 8 = 2

Relation d ’Euler en 4D

-v + f - a + s = 0

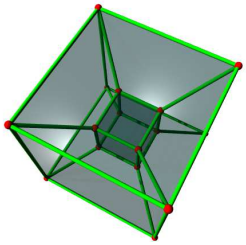
Le pentatope

-5 +10 –10 + 5 = 0

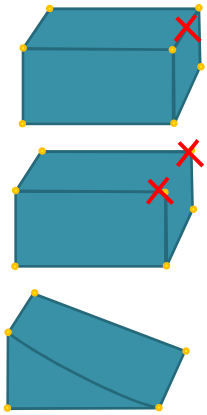


L’Hypercube

-8 +24 – 32 + 16 = 0



Polyèdres (Rel. d'Euler-Poincaré)



Card(F) – Card(A)+Card(S)= 2

6 - 12 + 8 = 2

KEF (-1) - (-1)

KEV - (-1) + (-1)

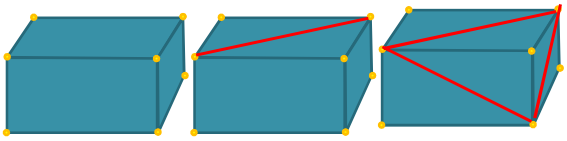
KEV - (-1) + (-1)

5 - 9 + 6 = 2

Triangulation

MEF (+1) - (+1)

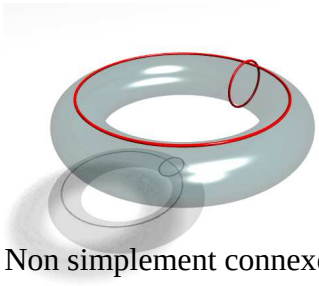
…



12 - 18 + 8 = 2

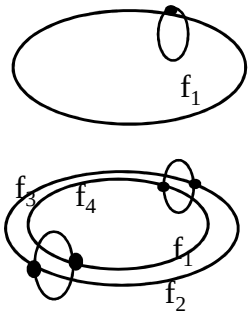
Exemples : Blender, CGAL

Les trous, le tore



Non simplement connexe

2 familles de cycles non réductibles et indépendants



f – a + s = 2

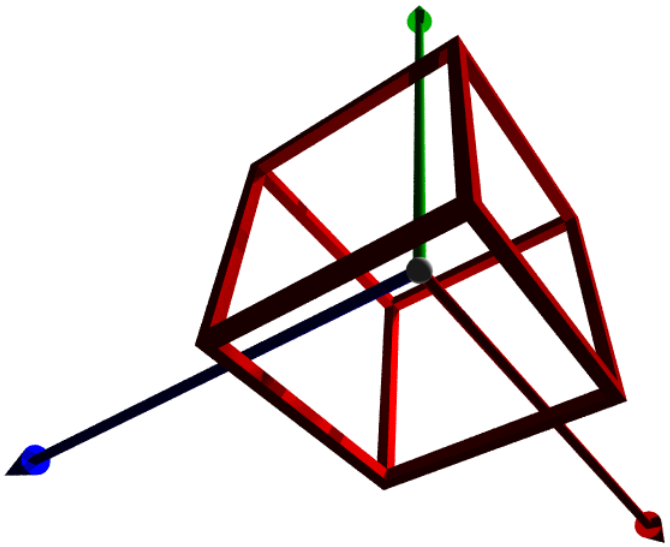
4 - 8 + 4 = 0

f – a + s = 2 (cc - h)

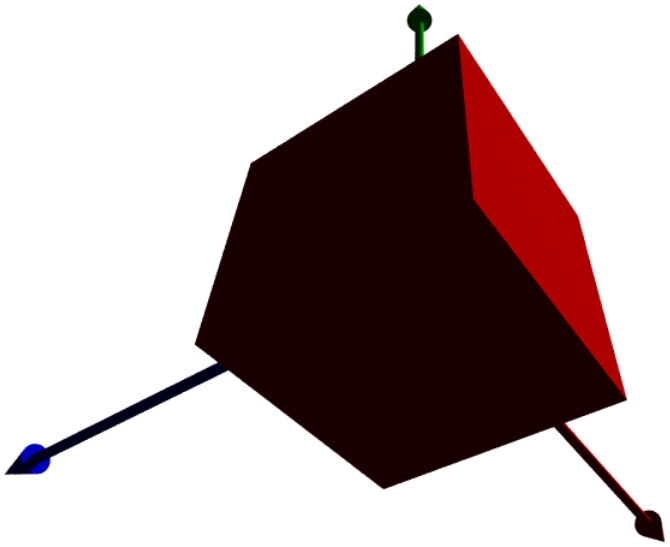
4 - 8 + 4 = 2 (1 - 1)

Avec CC : nombre de composantes connexes (arc) et H : nombre de trous (Hole)

Savez-vous compter les trous ?



Savez-vous compter les trous ?

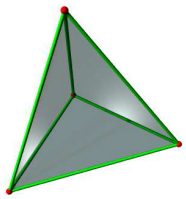


Triangulations avec trou (card)

Relation d'Euler :

$t-a+s=2(cc-h)$

4-6+4=2



Triangulation :

$3t=2a''$

3*4=2*6

$$t = 2s-4(cc-h)$$

cc le nombre de composantes connexes
h le nombre de trous (hole)

4=2*4-4

Triangulations avec trous ?

Cardinalités :

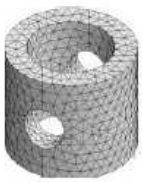
1. S = 924 ; F = 1856
2. S = 925 ; F = 1850
3. S = 919 ; F = 1842

Triangulations :

A.

B.

C.



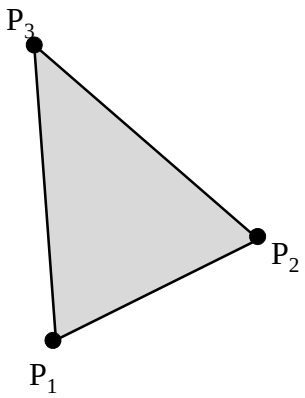
Trouvez la correspondance : 1,2,3➔A,B,C ?

Interrogation et modifications du modèle (2D/3D)

- Aire d'un polygone? Volume polyèdre ?
- Un point (x,y) est-il à l 'intérieur ou l 'extérieur du polygone (du polyèdre) ?
- Intersection d 'une droite avec le polyèdre ?
- ...

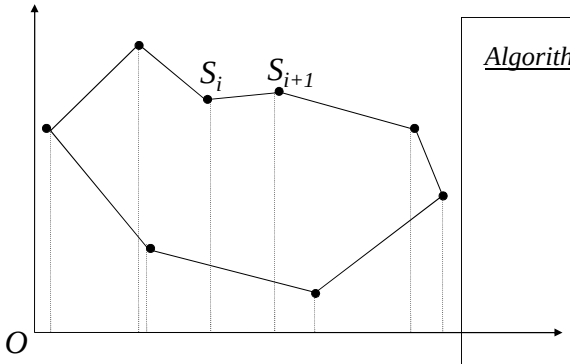
- Opérations booléennes entre des Brep

Aire d'un triangle ?



Calcul de l ’aire, du volume ?

Approche pragmatique



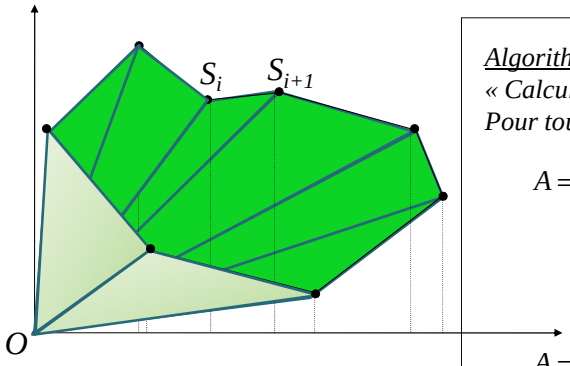
Extension au 3D ?

Algorithme :

Calcul de l ’aire, du volume

Approche pragmatique

Simplexe : triangle, tétraèdre
Polygone, Polyèdres



Extension au 3D ?

Algorithme :
« Calcul » du point O, A=0
Pour toutes les sommets du poly

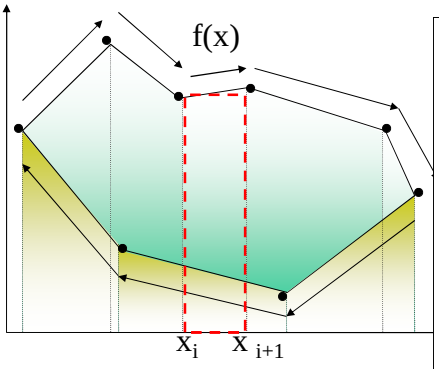
$$A = A + \frac{1}{2}\overrightarrow{OS_i} \Lambda \overrightarrow{S_i S_{i+1}}$$

$$A = \frac{1}{2}\sum_{i=0}^{n-1}(x_i y_{i+1} - x_{i+1} y_i)$$

Calcul de l 'aire

Approche triviale : représentation algébrique explicite : y = A x + B

$$\sum_{i=1}^{n} \int_{x_i}^{x_{i+1}} f(x)$$



Exemple cas linéaire :

$f(x)= (x-x_i)(y_{i+1}-y_i)/(x_{i+1}-x_i)+ y_i$

Ou encore

$f(x)=Ax+B$

avec $A=(y_{i+1}-y_i)/(x_{i+1}-x_i)$

et $B=(x_{i+1}y_i-x_iy_{i+1})/(x_{i+1}-x_i)$

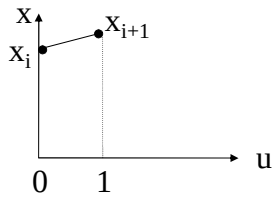
$$\int_{x_i}^{x_{i+1}} (Ax+B)dx =$$

$$\frac{1}{2}(y_{i+1}+y_i)(x_{i+1}-x_i)$$

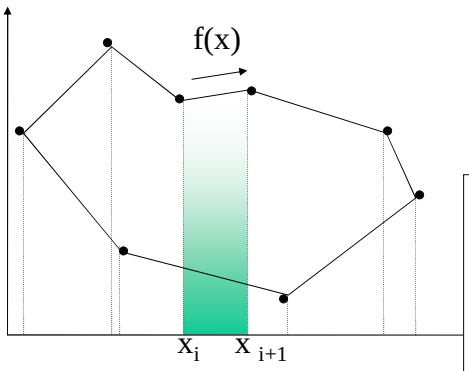
Calcul de l'Aire

Avec changement de variable : représentation paramétrique

$$\int_{x=x_i}^{x_{i+1}} f(x)dx = \int_{u=0}^{1} f(t(u))t'(u)du$$



x = t(u), u dans [0:1]



Exemple cas linéaire :

$t(u) = ux_{i+1} +(1-u)x_i$

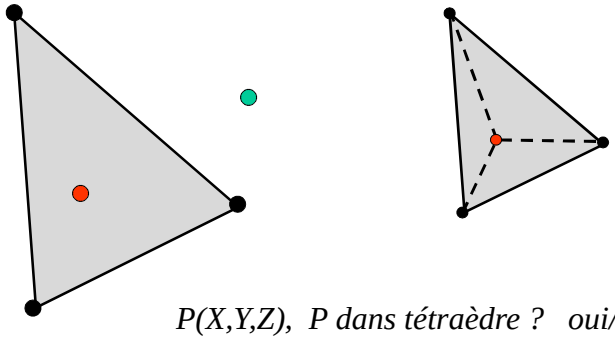
$f(t(u))= uy_{i+1} +(1-u)y_i$

$$\int (uy_{i+1}+(1-u)y_i)(x_{i+1}-x_i)du =$$

$$\frac{1}{2}(y_{i-1}+y_i)(x_{i+1}-x_i)$$

Point dans simplexe ?

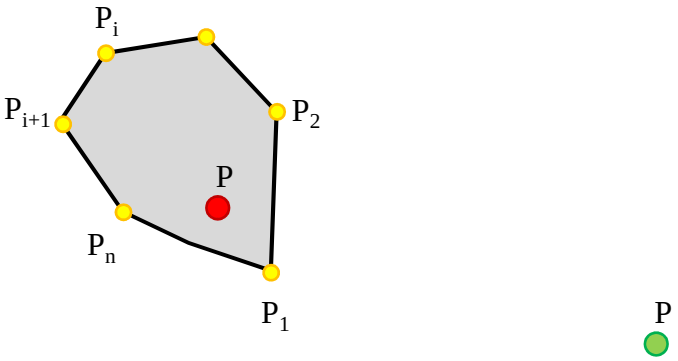
P(X,Y), P dans triangle ? oui/non



P(X,Y,Z), P dans tétraèdre ? oui/non

P dans convexe (2D,3D)? oui/non

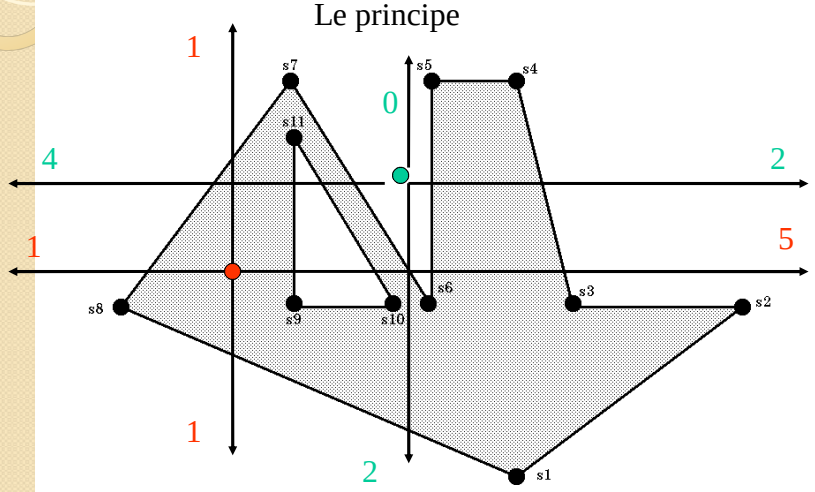
Point dans convexe ?



Complexité de l'algorithme ?

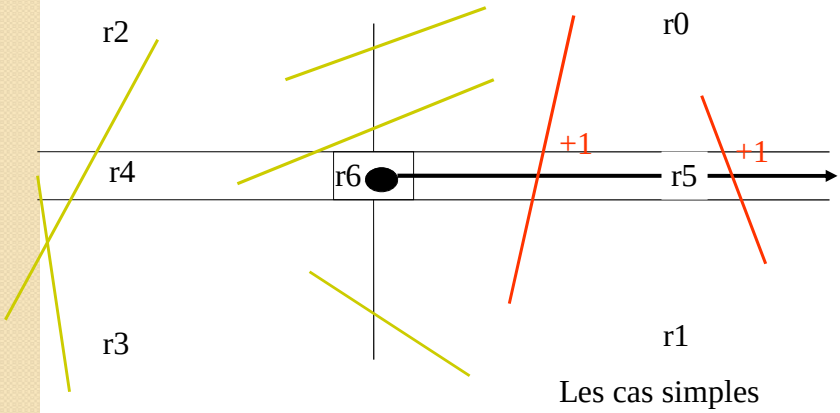
Point dans polygone ?

Le principe



Point dans polygone ?

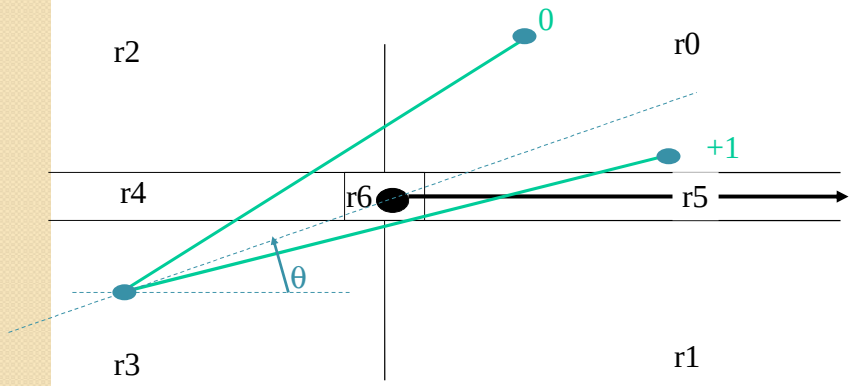
L ’algorithme



Les cas simples

Point dans polygone ?

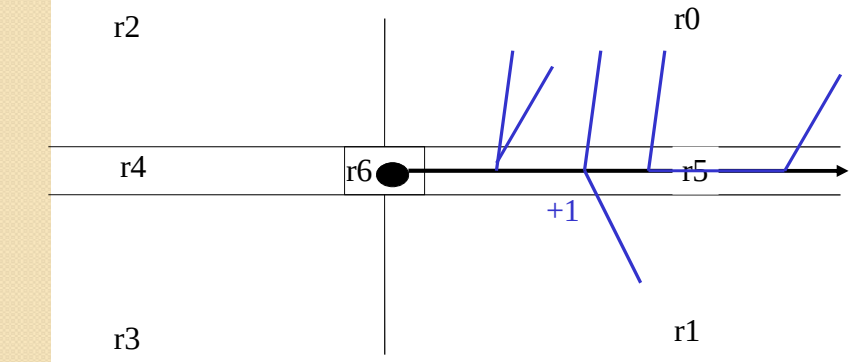
L ’algorithme



Singularité simples

Point dans polygone ?

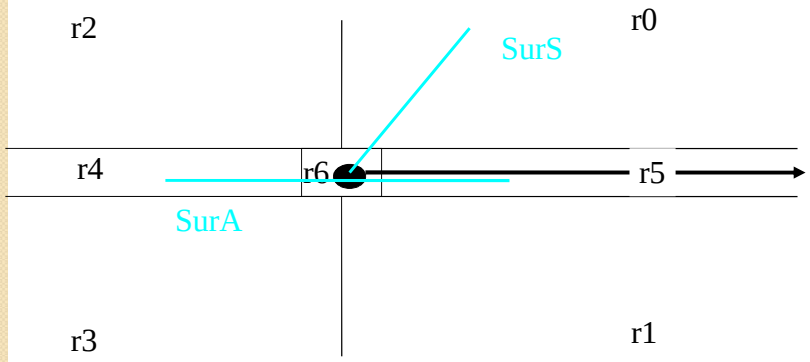
L ’algorithme



Singularité compliquée

Point dans polygone ?

L ’algorithme



Cas terminaux

Point dans polygone ?

L ’algorithme

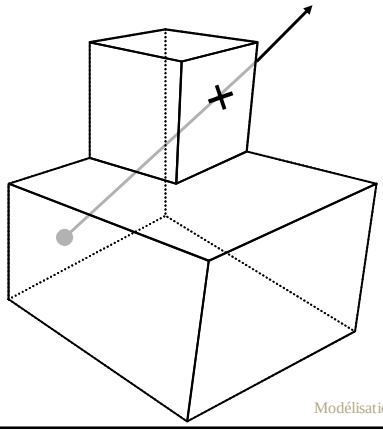
	r0	r1	r2	r3	r4	r5	r6
r0	0	+1	0		0		SurS
r1	+1	0		0	0		SurS
r2	0		0	0	0		SurS
r3		0	0	0	0		SurS
r4	0	0	0	0	0	SurA	SurS
r5					SurA		SurS
r6	SurS	SurS	SurS	SurS	SurS	SurS	SurS

Test de l ’angle

Parcours structure

Point dans polyèdre ?

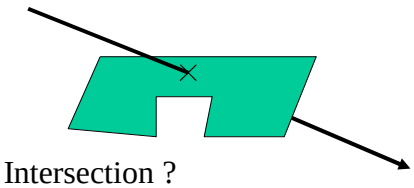
Même principe : on compte le nombre d'intersections



Intersection avec une droite ?

Cas 3D

METHODE : Pour toutes les faces du polyèdre…



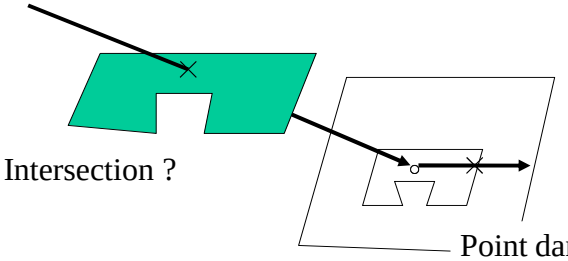
Intersection ?

Intersection avec une droite

Cas 3D

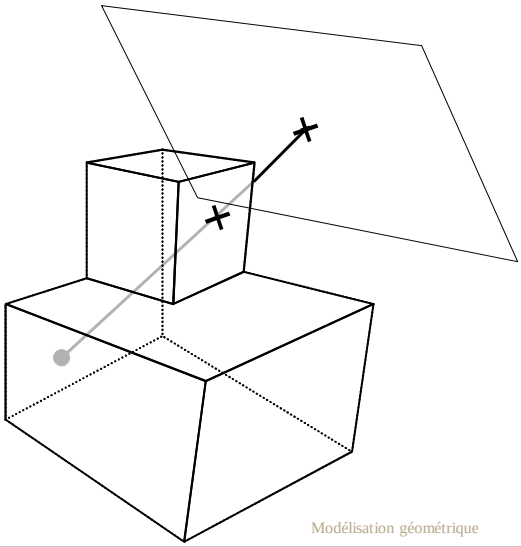
METHODE : Pour toutes les faces du polyèdre…

Projeter le polygone dans le plan perpendiculaire à la droite



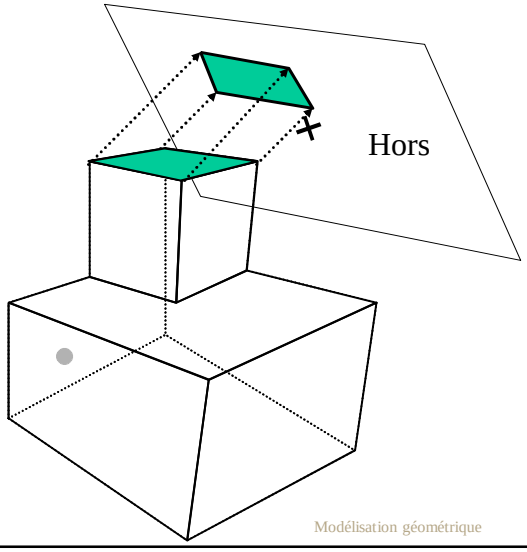
Point dans polyèdre ?

- Exemple



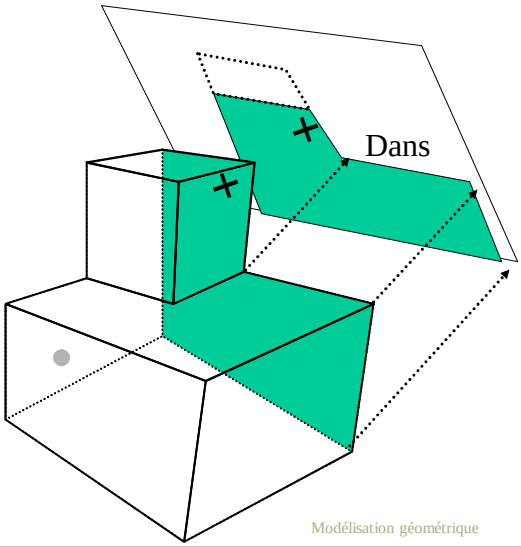
Point dans polyèdre ?

- Exemple



Point dans polyèdre ?

- Exemple



Opérations booléennes

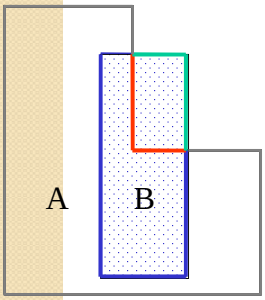
Fr(A), Fr(B)

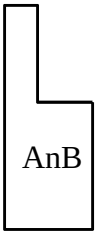
Fr(AnB) =

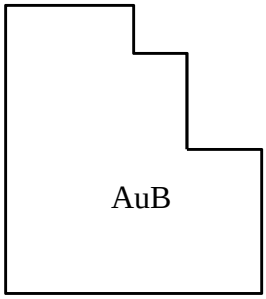
Fr(B)dans A
Fr(A)dans B

Fr(AuB) =

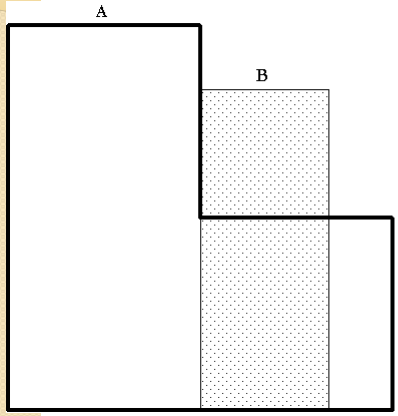
Fr(A) hors B
Fr(B)hors A

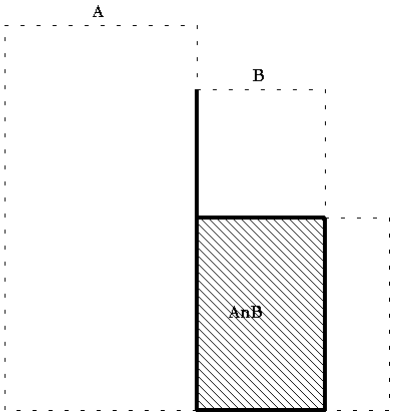




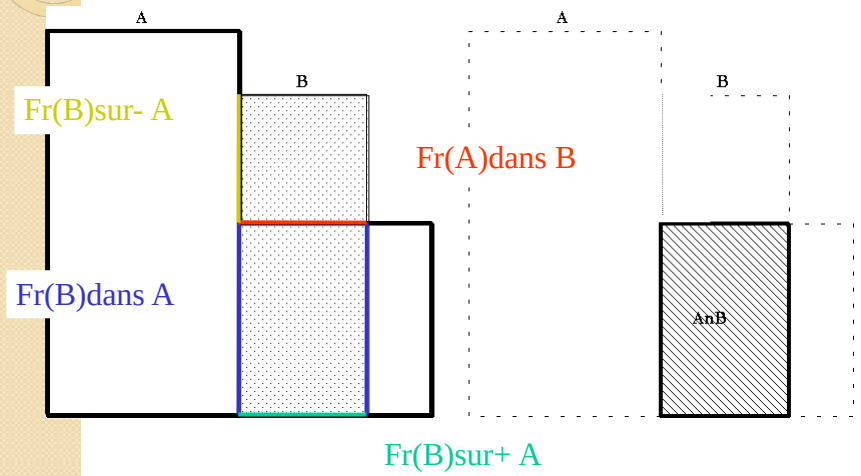


Opérations booléennes régularisées





Opérations booléennes régularisées



Opérations booléennes régularisées

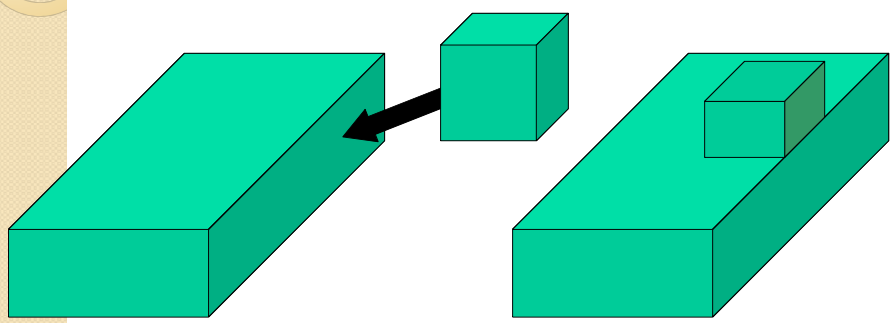
Fr(AnB) = Fr(A) dans B + Fr(B) dans A + Fr(B) sur + A

Fr(AuB) = Fr(A) hors B + Fr(B) hors A + Fr(B) sur +A

Fr(A-B) = Fr(A) hors B + Fr(B) dans A + Fr(B) sur - A

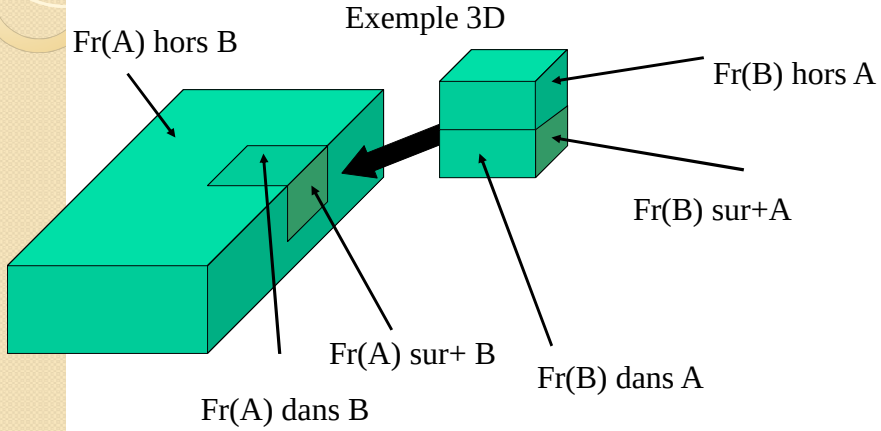
Opérations booléennes

Exemple 3D



Opérations booléennes

Exemple 3D



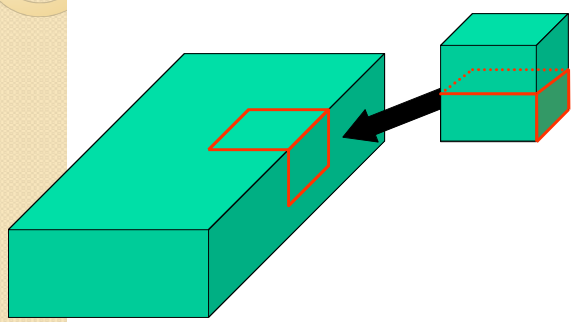
Opérations booléennes

Schéma de l'algorithme 2D et 3D

- 1. Calcul des intersections
- 2. Classification des parties de frontière
- 3. Suppression des parties inutiles
- 4. Collage des parties restantes

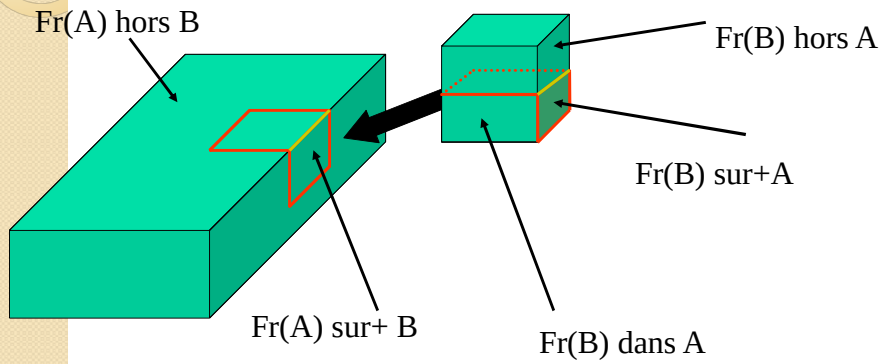
Opérations booléennes

Étape 1 : Arêtes = intersection des faces



Opérations booléennes

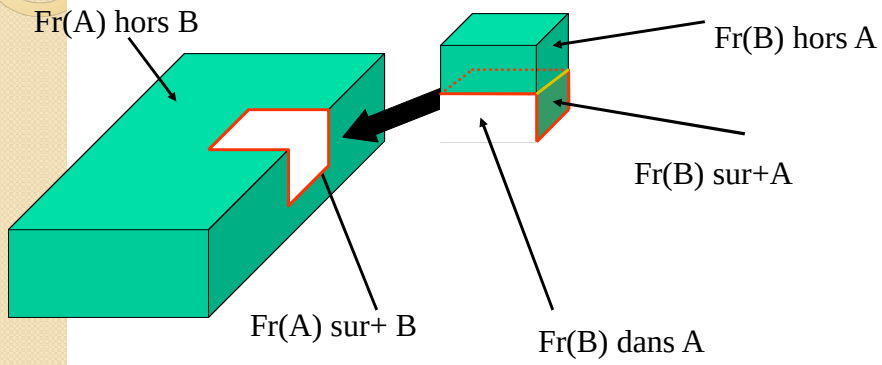
Étape 2 : Classification des portions de frontière



Fr(AuB) = Fr(A) hors B + Fr(B) hors A + Fr(B) sur +A

Opérations booléennes

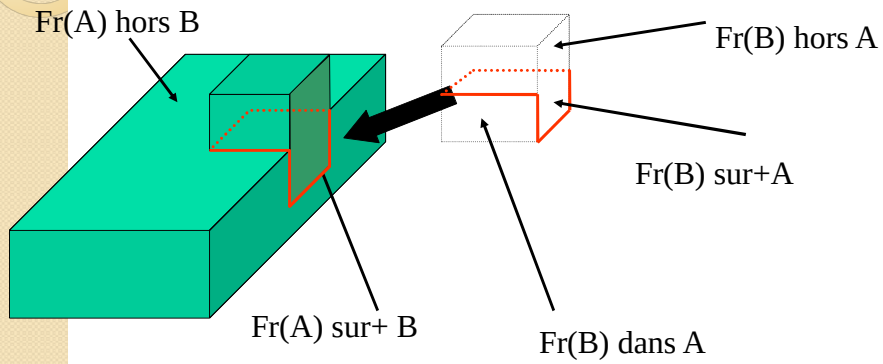
Étape 3 : Suppression des portions de frontière



Fr(AuB) = Fr(A) hors B + Fr(B) hors A + Fr(B) sur +A

Opérations booléennes

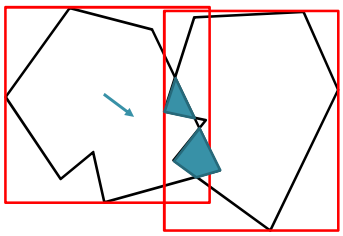
Étape 4 : Collage des portions de frontière



Fr(AuB) = Fr(A) hors B + Fr(B) hors A + Fr(B) sur +A

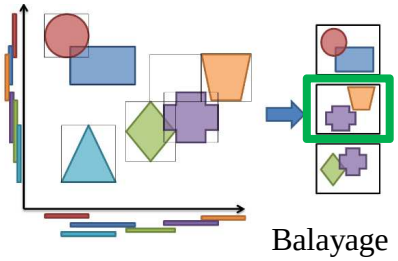
Intersection de 2 polygones

- **Détecter** : englobants & structures
- **Calculer**/estimer : C= AnB
- **Éviter** (trajectoire) : somme de Minkowski



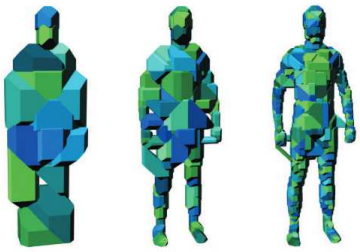
Détecter l'intersection

Volume englobant



Balayage

Hiérarchie

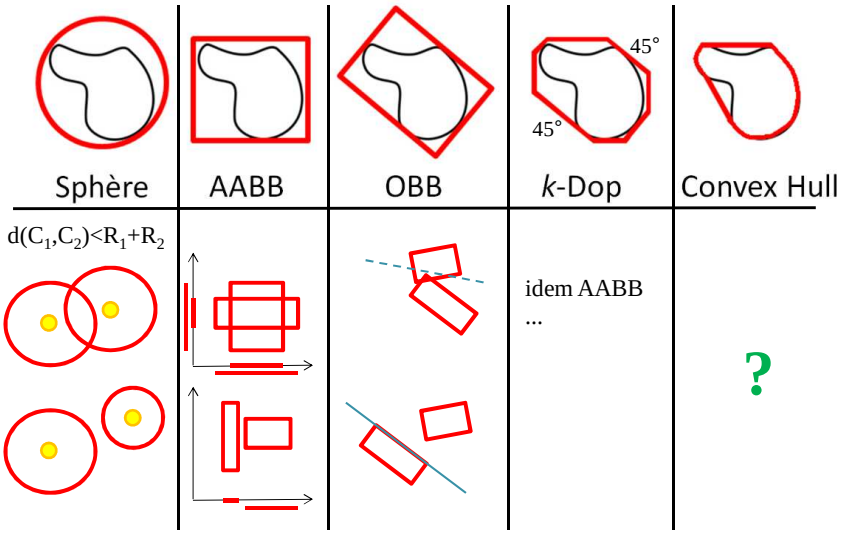


Structures d'accès / subdivision spatiale**

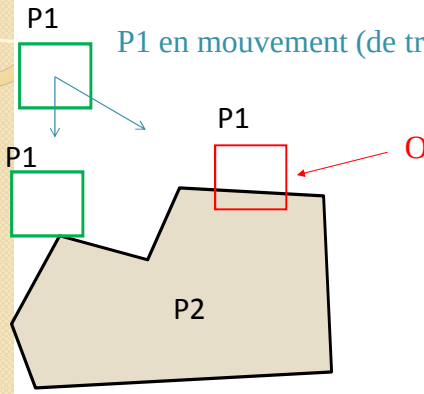
*Q.Avril

**Autre approche ➔que l'on verra plus tard (algorithmique géométrique)

Volumes englobant



Éviter les « Collisions »

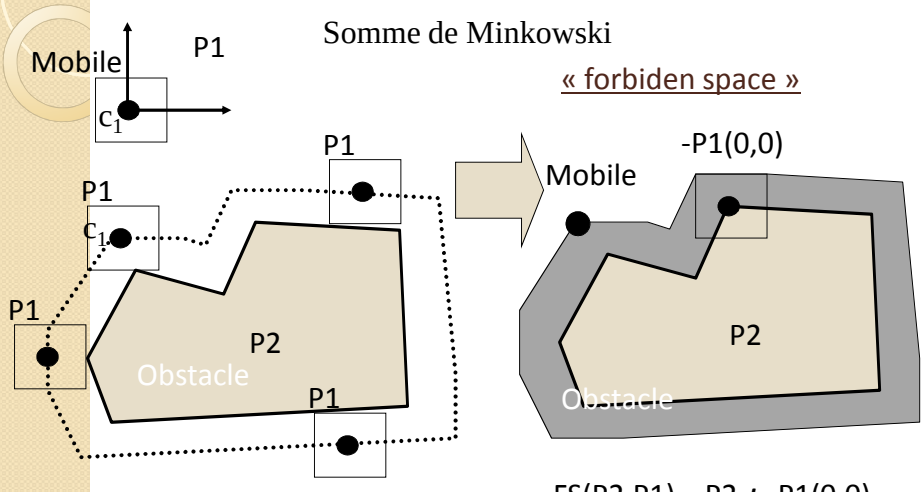


P2 immobile = obstacle

Zone sans collision

Somme de Minkowski

« forbiden space »

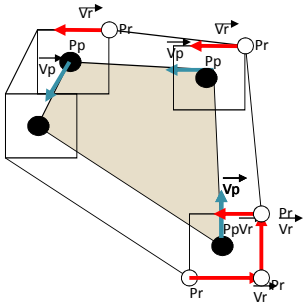


FS(P2,P1) = P2 + -P1(0,0)

Intersection P1 n P2 =0 ? ⇔ Point « c_1 » hors FS(P2,P1) ?

Somme de Minkowski

Polygones convexes



$\overrightarrow{Vr}$ = *Pr SUIV[Pr]*

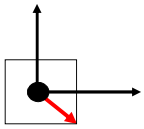
$\overrightarrow{Vp}$ = *Pp SUIV[Pp]*

Si(PrV(Vr, Vp)> 0)Alors

Pr = SUIV[Pr]
FS = FS u { Pp + Pr }

Sinon ***Pp = SUIV[Pp]***

FS = FS u { Pp + Pr }



VSi

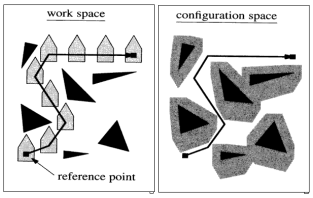
Les sommets du polygone P1 ⊕ P2 = sommets de P2 + VSi

Applications : jeux, robotique…

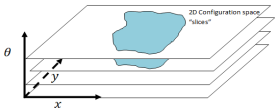
(Motion planning)

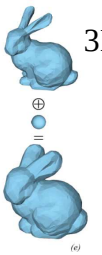
translation

2D : jeux vidéo

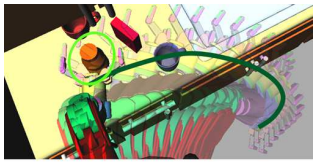


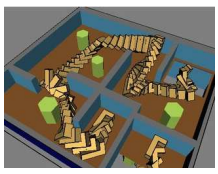
rotation



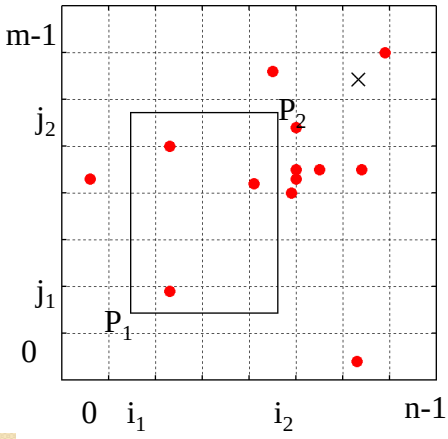


3D : Robotique





La grille



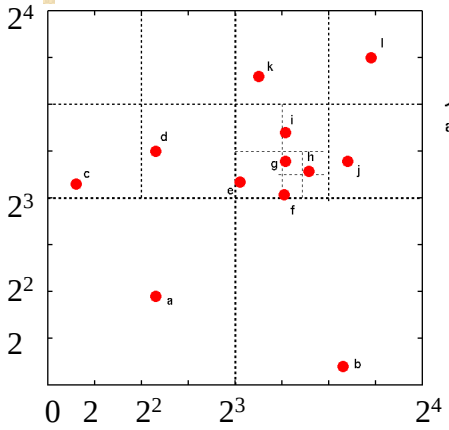
Requête boite $B(P_1,P_2)$
$P_1=(x_1,y_1)$, $P_2=(x_2,y_2)$

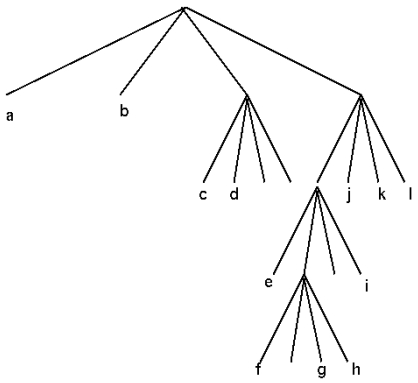
$i_1=n*(x_1-x_{min})/(x_{max}-x_{min})$
$j_1=m*(y_1-y_{min})/(y_{max}-y_{min})$

$i_2=n*(x_2-x_{min})/(x_{max}-x_{min})$
$j_2=m*(y_2-y_{min})/(y_{max}-y_{min})$

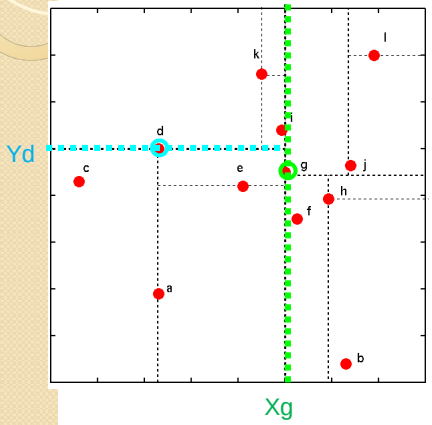
Cellules de i_1 à i_2 et de j_1 à j_2

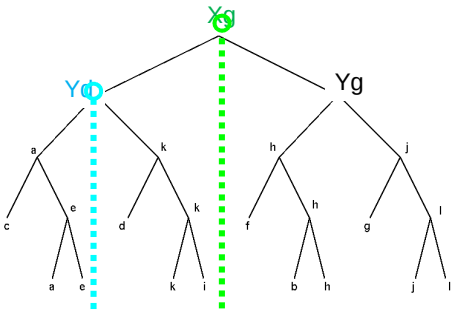
Le Quadtree



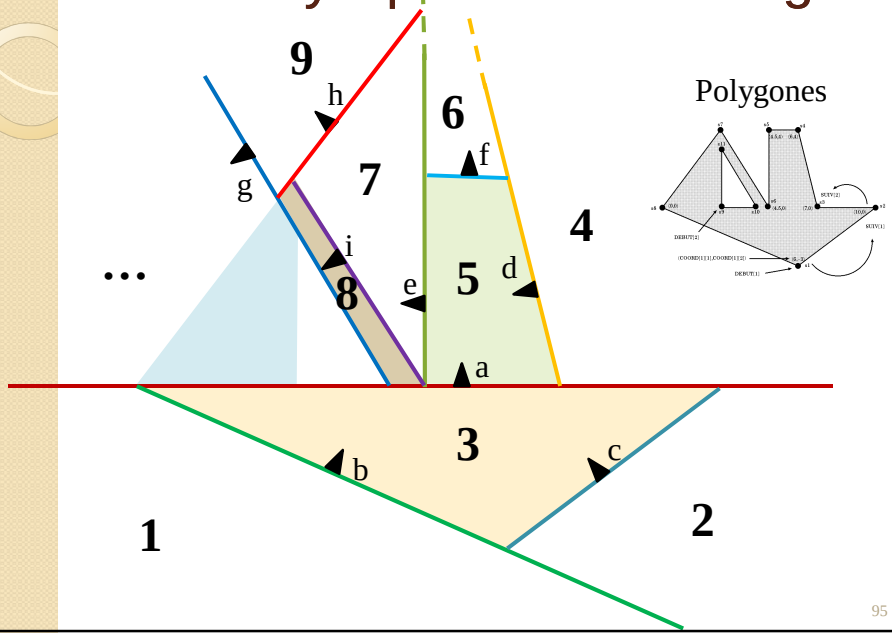


Le KDtree



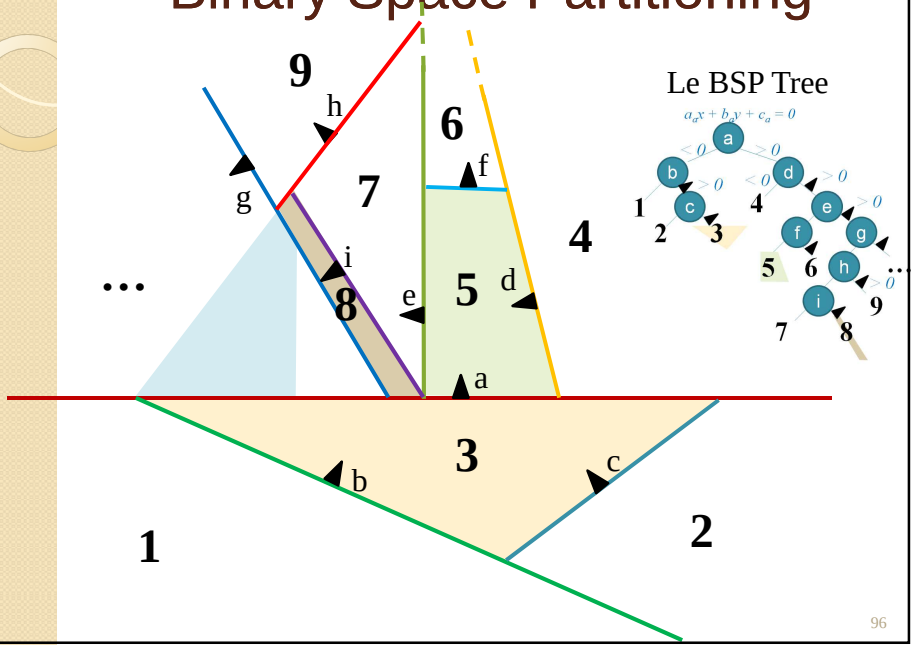


Binary Space Partitioning



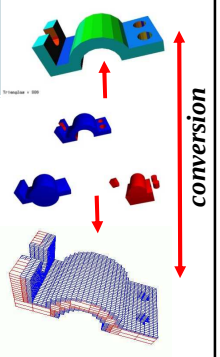
Polygones

Binary Space Partitioning



Conclusion

	Domaine	Complétude	Interaction	Visual.
Brep	large	OK	difficile	
CSG	large	incomplet	naturelle	
Enum	large	précision	laborieuse	



Bibliographie

- Introduction à la modélisation et à l'algorithmique géométrique Olivier STAB
  - *http://people.mines-paristech.fr/olivier.stab/geomodeling.pdf*
- Geometric Modeling
  Michael MORTENSON -*Wiley*
- Computer Graphics, principles and practice
  Foley, Van Dam, Feiner, Hughes - *Addison Wesley*
- Advanced Animation and Rendering Techniques,
  Alan & Mark Watt, *Addison Wesley*
- Modélisation et construction de surfaces pour la CFAO,
  J.C. Léon, *Hermes*
- Géométrie discrète en analyse d 'images
  J.M Chassery & A. Montanvert - *Hermes*