

# Deep-Learning: Recurrent Neural Networks (RNN)

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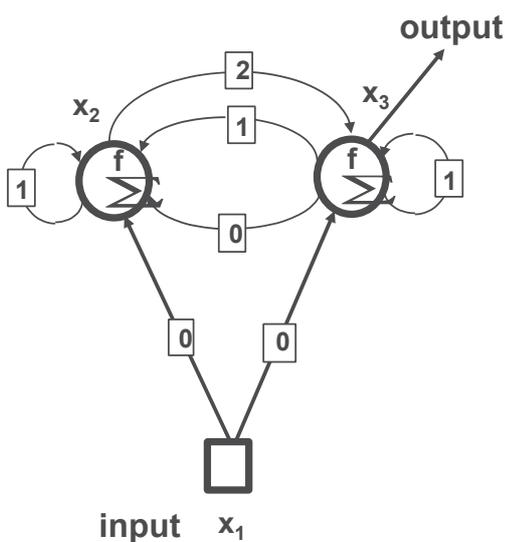
## Acknowledgements

During preparation of these slides, I got inspiration and borrowed some slide content from several sources, in particular:

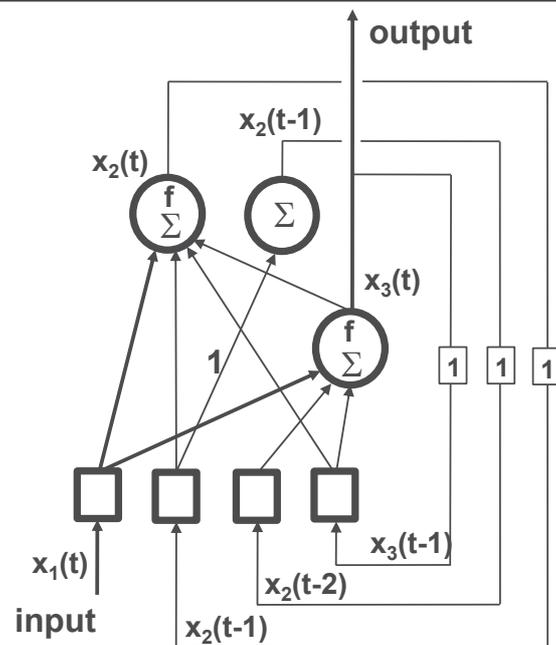
- Fei-Fei Li + J.Johnson + S.Yeung: slides on “*Recurrent Neural Networks*” from the “*Convolutional Neural Networks for Visual Recognition*” course at Stanford  
[http://cs231n.stanford.edu/slides/2019/cs231n\\_2019\\_lecture10.pdf](http://cs231n.stanford.edu/slides/2019/cs231n_2019_lecture10.pdf)
- Yingyu Liang: slides on “*Recurrent Neural Networks*” from the “*Deep Learning Basics*” course at Princeton  
[https://www.cs.princeton.edu/courses/archive/spring16/cos495/slides/DL\\_lecture9\\_RNN.pdf](https://www.cs.princeton.edu/courses/archive/spring16/cos495/slides/DL_lecture9_RNN.pdf)
- Arun Mallya: slides “*Introduction to RNNs*” from the “*Trends in Deep Learning and Recognition*” course of Svetlana LAZEBNIK at University of Illinois at Urbana-Champaign  
[http://slazebni.cs.illinois.edu/spring17/lec02\\_rnn.pdf](http://slazebni.cs.illinois.edu/spring17/lec02_rnn.pdf)
- Tingwu Wang: slides on “*Recurrent Neural Network*” for a course at University of Toronto  
[https://www.cs.toronto.edu/%7Etingwuwang/rnn\\_tutorial.pdf](https://www.cs.toronto.edu/%7Etingwuwang/rnn_tutorial.pdf)
- Christopher Olah: online tutorial “*Understanding LSTM Networks*”  
<https://colah.github.io/posts/2015-08-Understanding-LSTMs/>

- **Standard Recurrent Neural Networks**
- Training RNN: BackPropagation Through Time
- LSTM and GRU
- Applications of RNNs

## Recurrent Neural Networks (RNN)

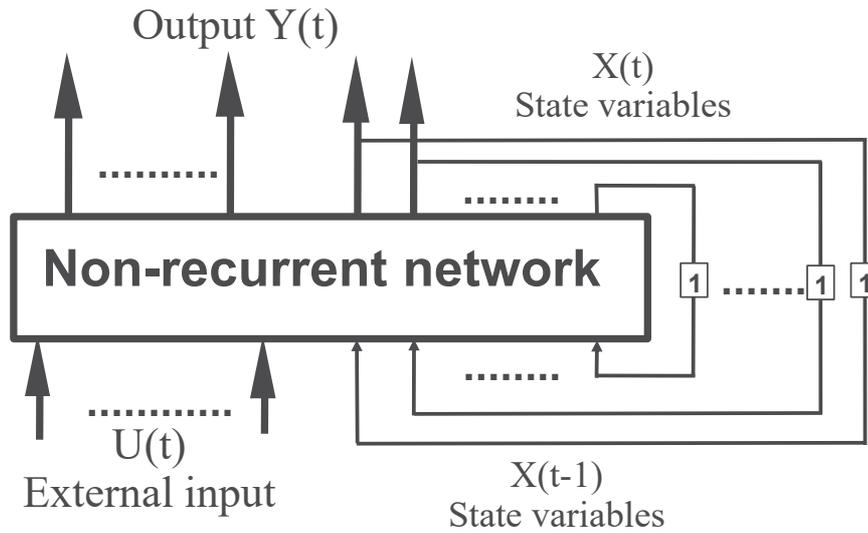


**Time-delay**  
for each connection

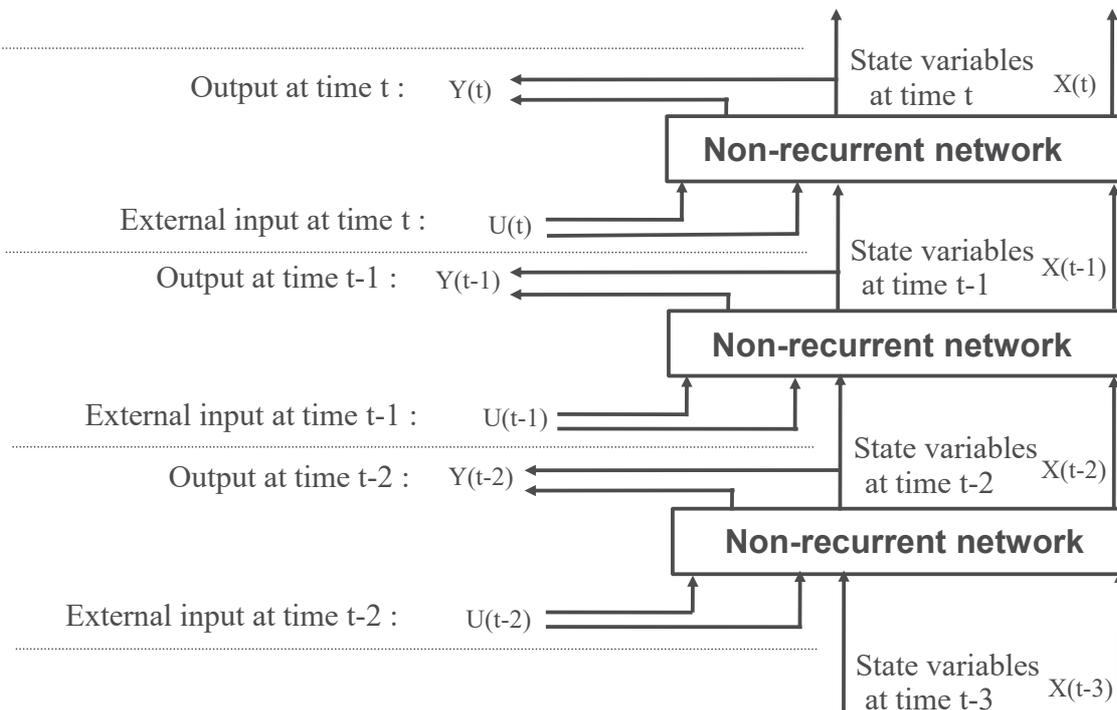


**Equivalent form**

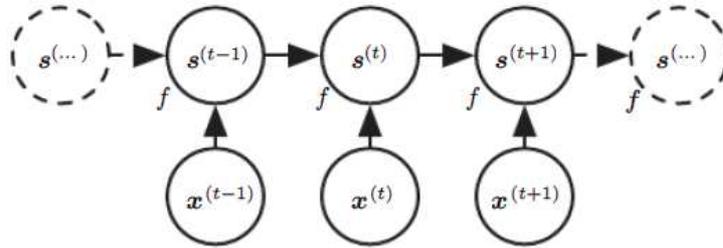
# Canonical form of RNN



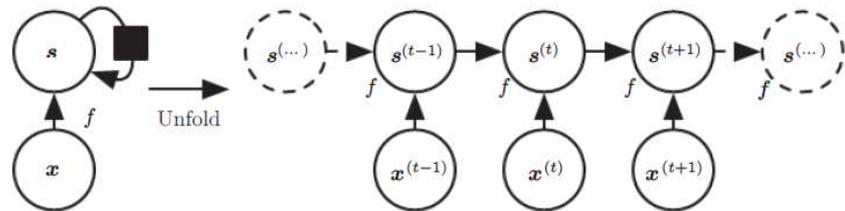
# Time unfolding of RNN



$$s^{(t+1)} = f(s^{(t)}, x^{(t+1)})$$



If using a Neural Net for  $f$ , this is **EXACTLY** a RNN!



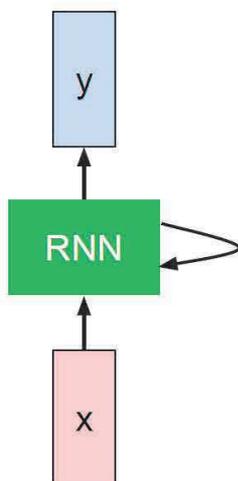
Figures from *Deep Learning*, Goodfellow, Bengio and Courville

State vector  $s \leftrightarrow$  vector  $h$  of hidden neurons

$$h_t = f_W(h_{t-1}, x_t)$$

new state / old state input vector at some time step

some function with parameters  $W$



$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

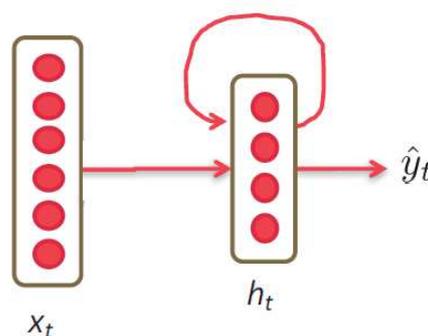
$$y_t = W_{hy}h_t$$

ou  $y_t = \text{softMax}(W_{hy}h_t)$

The hidden state  $s$  of the RNN builds a kind of lossy summary of the past

RNN totally adapted to processing SEQUENTIAL data (same computation formula applied at each time step, but modulated by the evolving “memory” contained in state  $s$ )

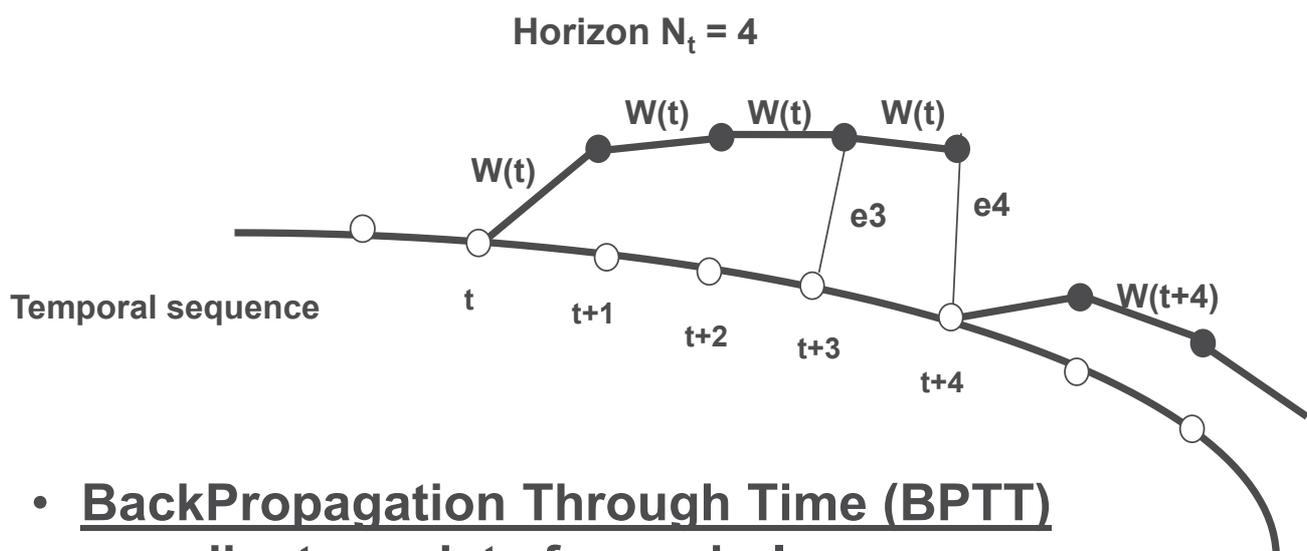
Universality of RNNs: any function computable by a Turing Machine can be computed by a finite-size RNN (Siegelmann and Sontag, 1995)



- As for MLP, main hyperparameter = size of hidden layer (=size of vector  $h$ )

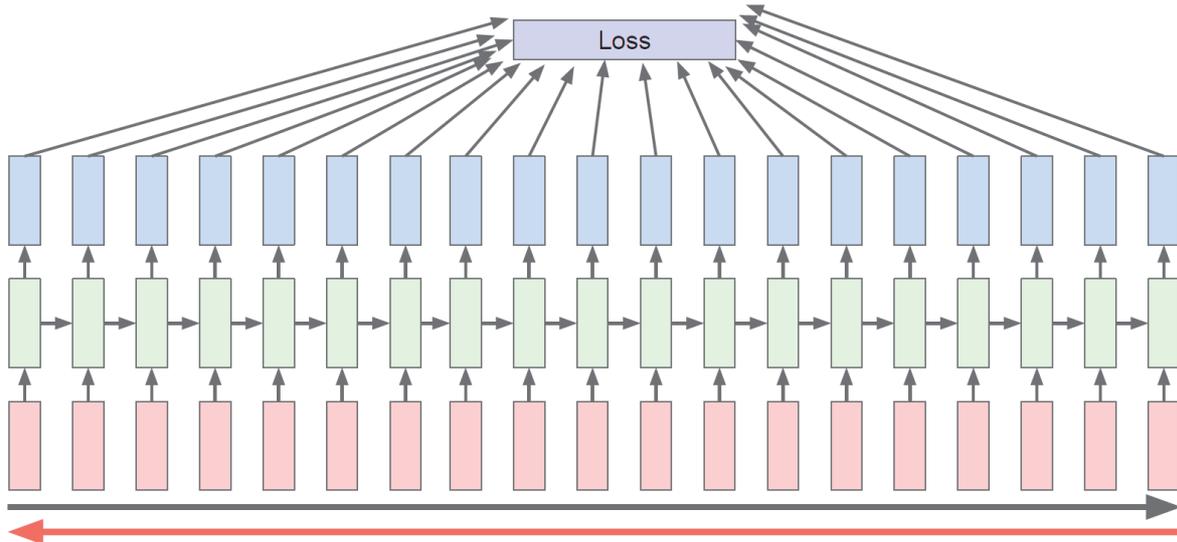
- Standard Recurrent Neural Networks
- **Training RNN: BackPropagation Through Time**
- LSTM and GRU
- Applications of RNNs

## RNN training



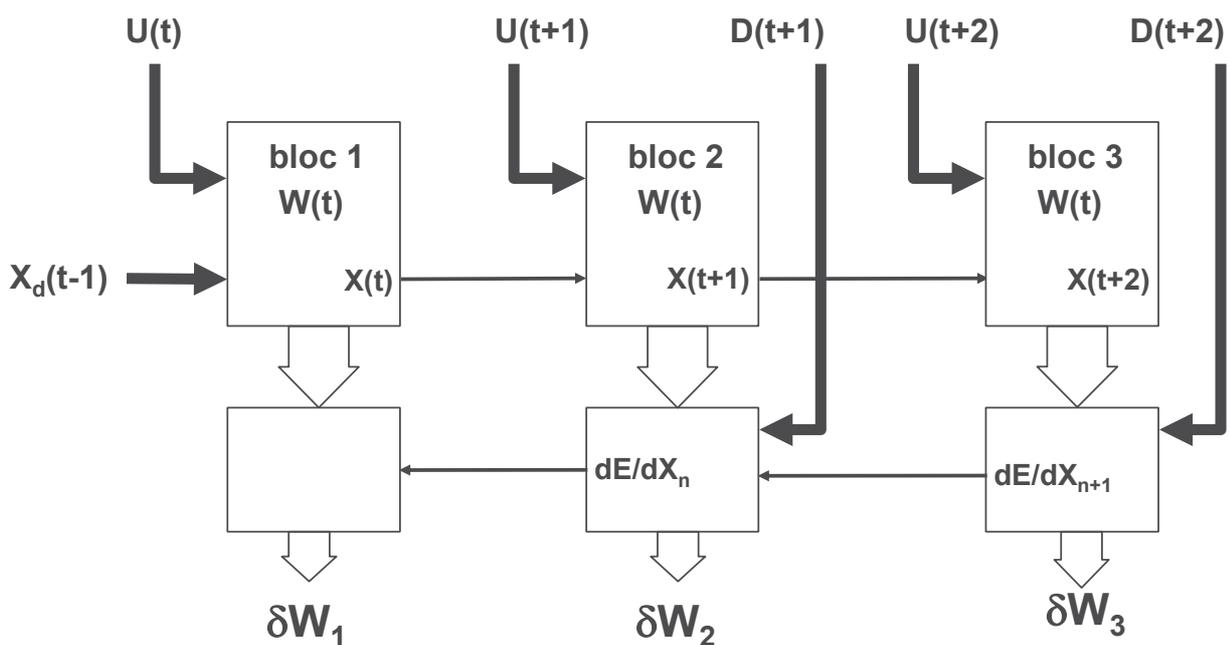
- **BackPropagation Through Time (BPTT)**  
gradients update for a whole sequence
- or Real Time Recurrent Learning (RTRL)  
gradients update for each frame in a sequence

# BackPropagation THROUGH TIME (BPTT)

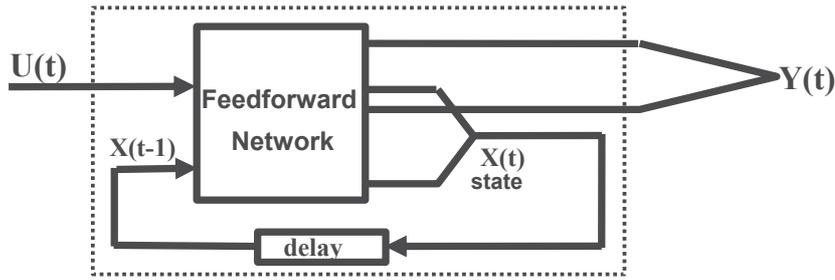


- Forward through entire sequence to compute SUM of losses at ALL (or part of) time steps
- Then backprop through ENTIRE sequence to compute gradients

## BPTT computation principle



$$\delta W = \delta W_1 + \delta W_2 + \delta W_3$$



$$W(t+N_t) = W(t) - \lambda \text{grad}_W(E) \text{ avec } E = \sum_{\tau} (Y_{\tau} - D_{\tau})^2$$

$$\frac{\partial E}{\partial W} = \sum_{t=1}^T \frac{\partial E_t}{\partial W} \quad \text{and} \quad \forall t, \frac{\partial E_t}{\partial W} = \frac{\partial E_t}{\partial Y_t} \frac{\partial Y_t}{\partial X_{t-1}} \frac{\partial X_{t-1}}{\partial W} \quad (\text{chain rule})$$

$$\frac{\partial X_t}{\partial W} = \sum_{k=1}^{t-1} \frac{\partial X_t}{\partial X_{t-k}} \frac{\partial X_{t-k}}{\partial W} \quad \frac{\partial X_t}{\partial X_{t-k}} = \prod_{j=1}^k \frac{\partial X_j}{\partial X_{j-1}} \quad \text{Jacobian matrix of the Feedforward net}$$

## Vanishing/exploding gradient problem

- If eigenvalues of Jacobian matrix  $>1$ , then gradients tend to EXPLODE  
 → Learning will never converge.
- Conversely, if eigenvalues of Jacobian matrix  $<1$ , then gradients tend to VANISH  
 → Error signals can only affect small time lags  
 → short-term memory.

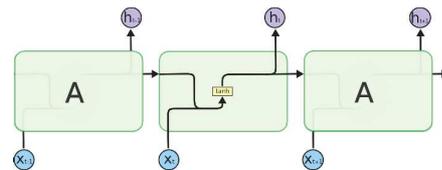
→ Possible solutions for exploding gradient:  
CLIPPING trick

→ Possible solutions for vanishing gradient:  
 – use ReLU instead of tanh  
 – *change what is inside the RNN!*

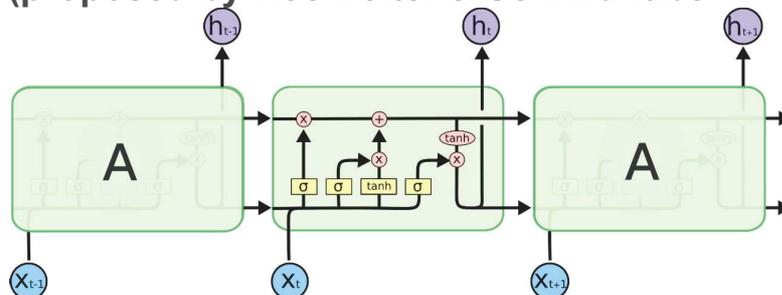
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## Long Short-Term Memory (LSTM)

Problem of *standard* RNNs =  
no actual LONG-TERM memory



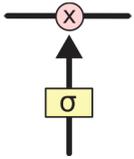
**LSTM = RNN variant for solving this issue**  
(proposed by Hochreiter & Schmidhuber in 1997)



[Figures from <https://colah.github.io/posts/2015-08-Understanding-LSTMs/>]

- **Key idea = use “gates” that modulate respective influences of input and memory**

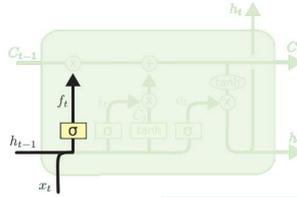
# LSTM gates



Gate = pointwise multiplication by  $\sigma$  in  $]0;1[$   
 → modulate between “let nothing through” and “let everything through”

- FORGET gate

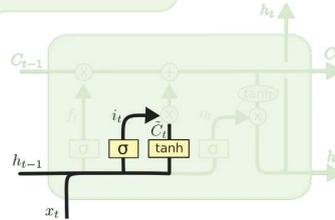
$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$



- INPUT gate

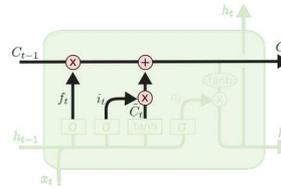
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$



→ next state = mix between pure memory or pure new

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$



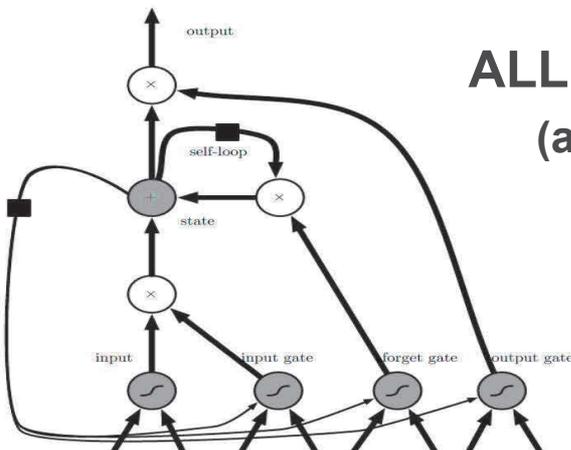
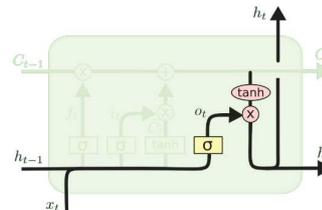
[Figures from <https://colah.github.io/posts/2015-08-Understanding-LSTMs/>]

# LSTM summary

- OUTPUT gate

$$o_t = \sigma(W_o [h_{t-1}, x_t] + b_o)$$

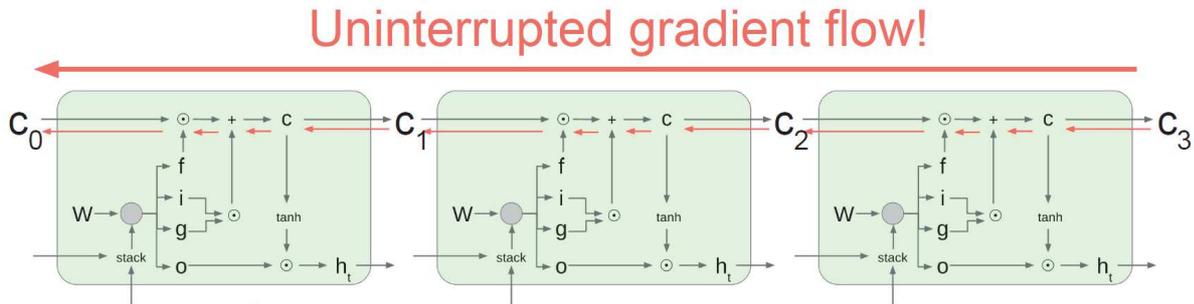
$$h_t = o_t * \tanh(C_t)$$



ALL weigths  $w_f$ ,  $w_i$ ,  $w_c$  and  $w_o$   
 (and biases) are LEARNT

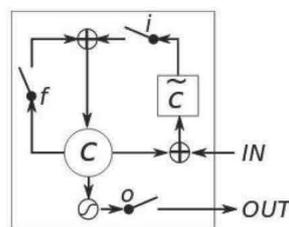
[Figure from *Deep Learning* book by I. Goodfellow, Y. Bengio & A. Courville]

# Why LSTM avoids vanishing gradients?



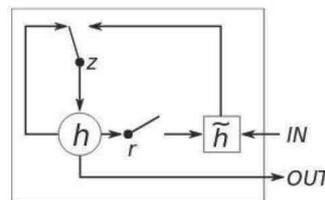
## Gated Recurrent Unit (GRU)

**Simplified variant of LSTM, with only 2 gates:  
a RESET gate & an UPDATE gate**  
(proposed by Cho, et al. in 2014)



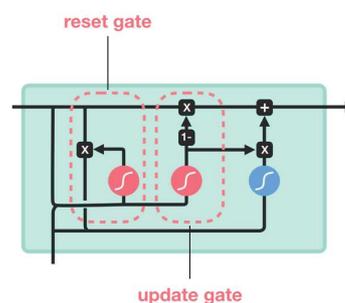
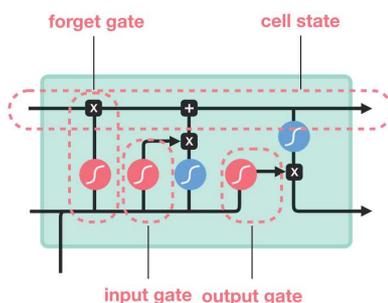
(a) Long Short-Term Memory

LSTM



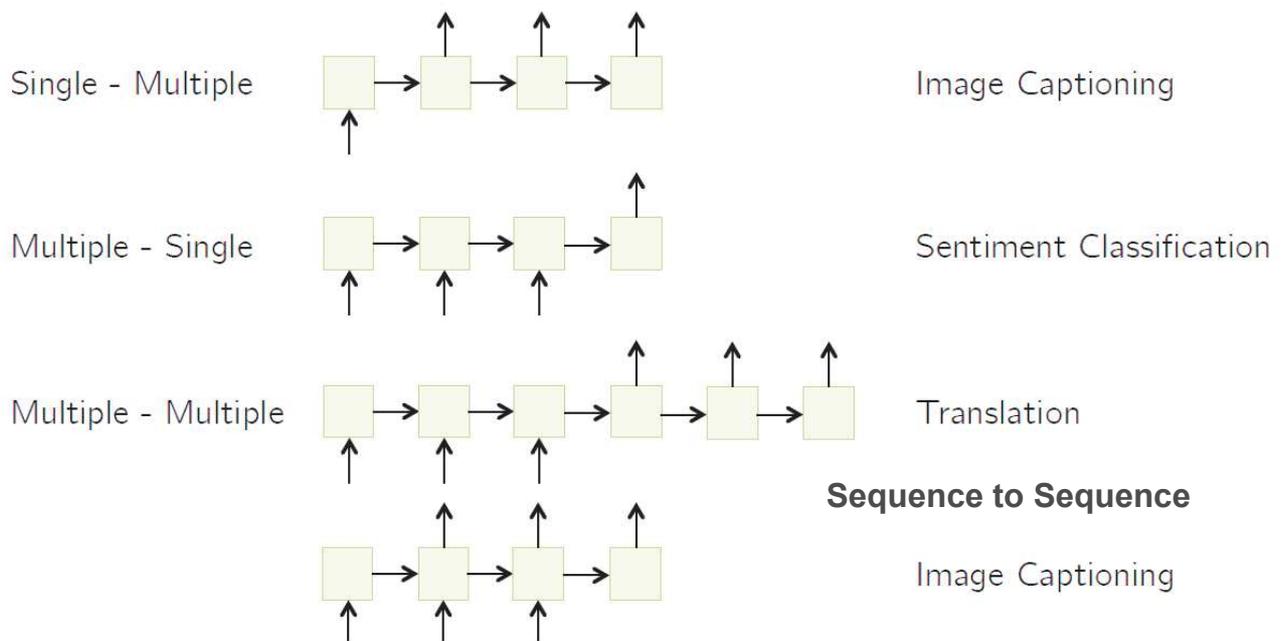
(b) Gated Recurrent Unit

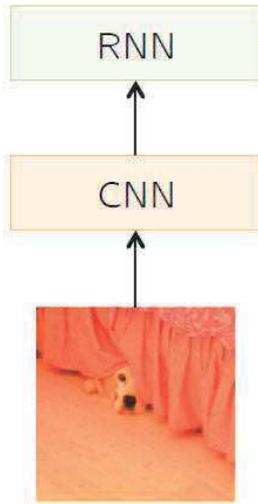
GRU



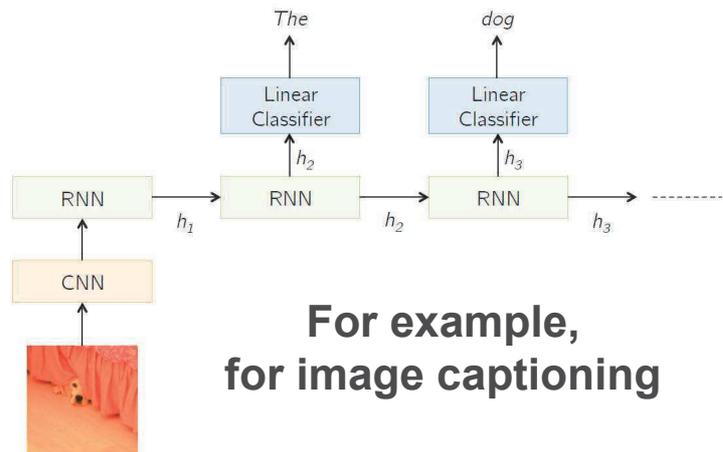
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## Typical usages of RNNs

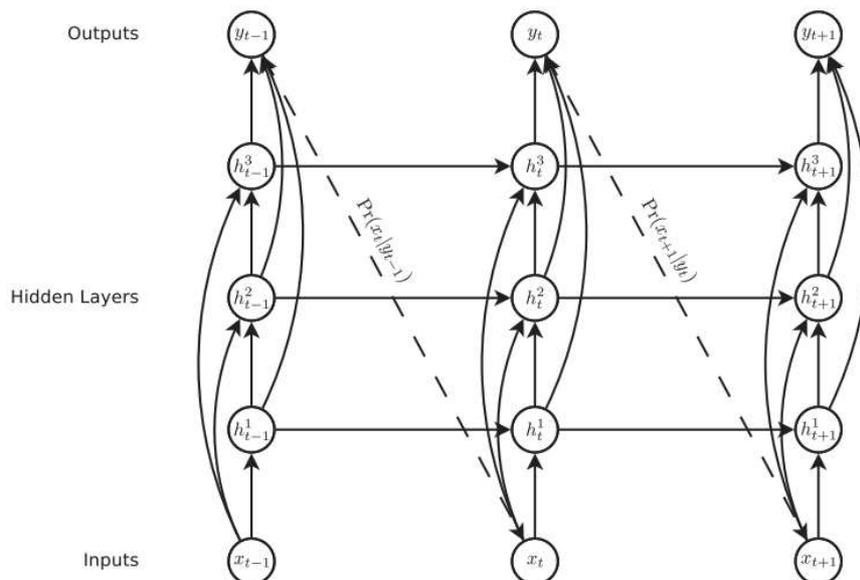




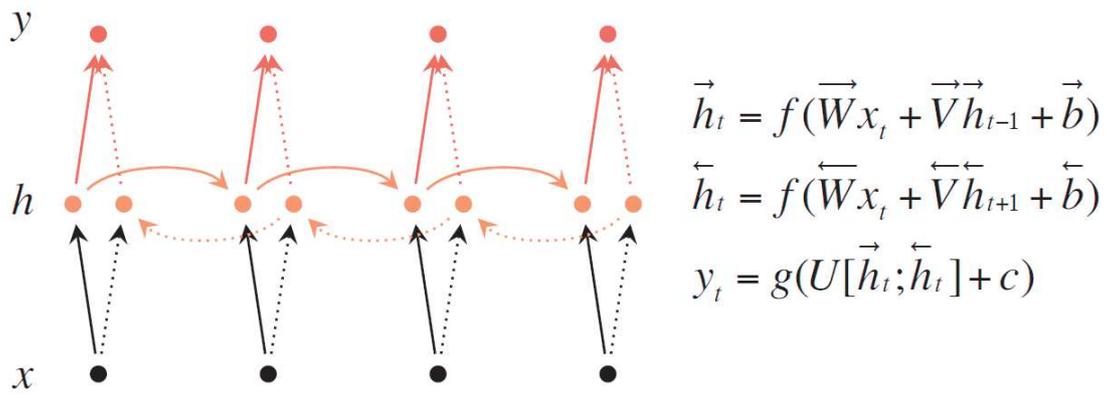
Input into RNN the features from last convolutional layer



For example, for image captioning

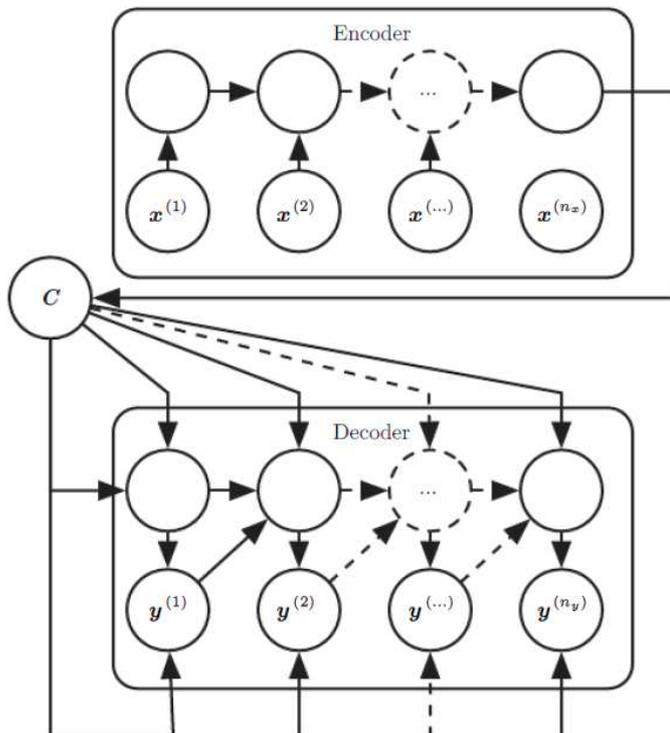


Several RNNs stacked (like layers in MLP)



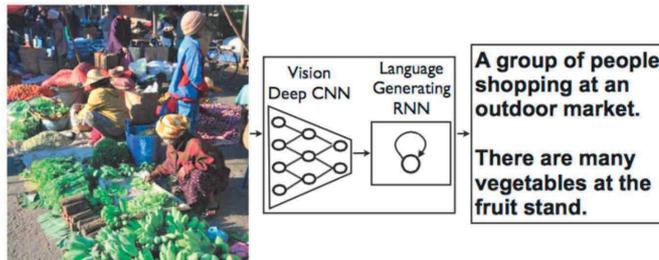
$h = [\vec{h}; \overleftarrow{h}]$  now represents (summarizes) the past and future around a single token.

(e.g. for offline classification of sequence of words)



## Wherever data is intrinsically SEQUENTIAL

- **Speech recognition**
- **Natural Language Processing (NLP)**
  - Machine-Translation
  - Image caption generator



- **Gesture recognition**
- **Music generation**
- ***Potentially any kind of time-series!!***

# Summary and perspectives on Recurrent Neural Networks

- For **SEQUENTIAL** data  
(speech, text, ..., gestures, ...)
- Impressive results in  
Natural Language Processing (in particular  
Automated Real-Time Translation)
- Training of standard RNNs can be tricky  
(vanishing gradient...)
- LSTM / GRU now more used than standard RNNs

**Any QUESTIONS ?**